Laser Cutting – Co-ordinate Geometry Trial #2

Introduction

This assessment requires you to apply co-ordinate geometry methods using a specific shape on a co-ordinate plane.

The quality of your discussion and reasoning will determine your overall grade.

- Show your calculations.
- Use appropriate mathematical statements.
- Clearly communicate your strategy and method at each stage of your solution.

Context

Metal is often cut by lasers, which give an identical and precise cut every time.

In this task you are asked to do some calculations which would allow a cutter to be properly programmed to cut a pair of triangles, with holes at their centres of gravity.

The actual cutter would have some units (millimetres in this case). You may omit them in your working.

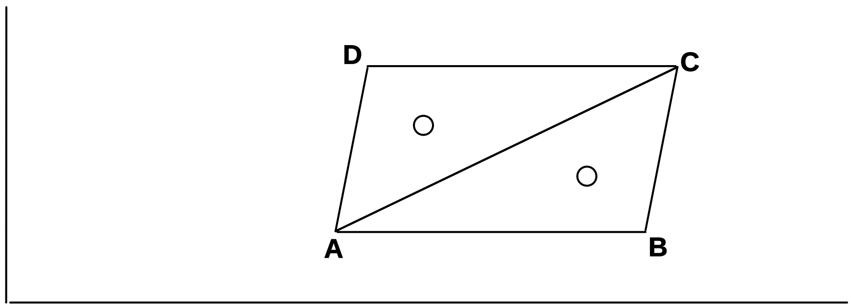
Centre of Gravity

The centre of gravity of a triangle is found by the intersection of its medians (a median is the line from a vertex to the midpoint of the opposite side).

Task

The initial triangle **ABC** has vertices **A** (10, 24), **B** (140, 24) and **C** (153, 180).

D is a point so that triangle **ACD** is exactly the same shape and shares the side **AC** with **ABC**.



- find the equation of the lines required to cut **ABC** so that they can be programmed in.
- calculate the total distance of the cuts to be made to cut out **ABC**.
- calculate the co-ordinates of the centres of gravity of **ABC**.
- repeat the steps above for **ACD**.

Formula: $y - y_1 = m\,(x - x_1)$

Solutions to Laser Cutting:

$m_{AB} = \frac{24-24}{140-10} = \frac{0}{130} = 0$ $\quad$ $y - 24 = 0(x - 140)$ gives $\boldsymbol{y = 24}$

$m_{BC} = \frac{180-24}{153-140} = \frac{156}{13} = 12$ $\quad$ $y - 24 = 12(x - 140)$ gives $\boldsymbol{y = 12x - 1656}$

$m_{AC} = \frac{180-24}{153-10} = \frac{156}{143} = \frac{12}{11}$ (= 1.0909) $\quad$ $y - 24 = \frac{12}{11}(x - 10)$ gives $\boldsymbol{y = \frac{12}{11}x + \frac{144}{11}}$

$|BC| = \sqrt{(153-140)^2 + (180-24)^2} = \sqrt{24505} = \mathbf{156.54}$ $\quad$ $|AB| = \mathbf{130}$

$|AC| = \sqrt{(153-10)^2 + (180-24)^2} = \sqrt{44785} = \mathbf{211.62}$

So the distance around ABC = 130 + 156.54 + 211.62 = **498.16**

To find the centre of gravity we need to find the medians, which means finding the midpoints. (NB: you only need to find two medians, so you only need two midpoints.)

Point M, midpoint of AB = $\left(\frac{10+140}{2}, \frac{24+24}{2}\right)$ = **(75, 24)**

Point N, midpoint of BC = $\left(\frac{140+153}{2}, \frac{24+180}{2}\right)$ = **(146.5, 102)**

Point O, midpoint of AC = $\left(\frac{10+153}{2}, \frac{24+180}{2}\right)$ = **(81.5, 102)**

Achieved *is obtained by showing three basic skills in co-ordinate geometry. While it cannot be put in black and white, a reasonable standard would be getting the lengths and midpoints (perhaps with an arithmetic error or two permitted) and at least one equation of a line.*

Now we need the slopes from those midpoints to the opposite vertex, and then plug them into a point to get the equation of the medians.

$m_{CM} = \frac{180-24}{153-75} = \frac{156}{78} = 2$

Median CM : $y - 180 = 2(x - 153) \Rightarrow y - 180 = 2x - 306$ $\quad$ $\Rightarrow \boldsymbol{y = 2x - 126}$

$m_{AN} = \frac{24-102}{10-146.5} = \frac{-78}{-136.5} = \frac{4}{7} = 0.5714$

Median AN : $y - 24\ \frac{4}{7}(x - 10) \Rightarrow y - 24 = \frac{4}{7}x - \frac{40}{7}$ $\quad$ $\Rightarrow \boldsymbol{y = \frac{4}{7}x + \frac{128}{7}}$

$m_{BO} = \frac{24-102}{140-81.5} = \frac{-78}{58.5} = \frac{-4}{3} = {}^{-}1.33333$

Median BO : $y - 24 = \frac{-4}{3}(x - 140) \Rightarrow y - 24 = \frac{-4}{3}x + 186\frac{2}{3}$ $\quad$ $\Rightarrow \boldsymbol{y = \frac{-4}{3}x + 210\frac{2}{3}}$

These median lines meet at the centre of gravity. Solving our simultaneous equation (by hand or on calculator) we get that it is at **(101, 76)**

Merit *would be to get most of the way to this solution, certainly needing to have the correct method, although a minor arithmetic error might be corrected on resubmission.*

We can use the parallel nature of ABCD to cut down the amount of work for triangle ACD.

The line AD is parallel to BC so its slope is 12 and it goes through A.

$y - 24 = 12(x - 10)$ gives **$y = 12x - 96$**

The line CD is parallel to AB so slope = 0 and goes through C, gives **$y = 180$**

AC is the same **$y = \frac{12}{11}x + \frac{144}{11}$**

The point D is where AD meets CD, which solves to give **D (23, 180)**

(NB: this can be done much more quickly by inspection of C being (x_B + 13, y_B + 156) and doing the same to A).

The distance around ACD is exactly the same as around ABC = **498.16**

To find the centre of gravity we can again use the symmetry of the parallelogram. (NB: again calculating it only needs two medians, but the third is shown for completeness.)

Median from D = median BO as it is a parallelogram : $\Rightarrow$ **$y = \frac{-4}{3}x + 210\frac{2}{3}$**

Point P, midpoint of DC = $(\frac{23+153}{2}, \frac{180+180}{2})$ = (88, 180)

By symmetry AP is parallel to CM, so the gradient = 2. Calculating $m_{AP} = \frac{180-24}{88-10} = \frac{156}{78} = 2$

DP is $y - 180 = 2(x - 88)$ which gives **$y = 2x + 4$**

Point Q, midpoint of AD = $(\frac{10+23}{2}, \frac{24+180}{2})$ = (15.5, 102)

By symmetry CQ || AN, so the gradient = $\frac{4}{7}$. Calculating $m_{CQ} = \frac{180-102}{153-15.5} = \frac{78}{136.5} = \frac{4}{7}$

CQ is $y - 180 = \frac{4}{7}(x - 153) \Rightarrow y - 24 = \frac{4}{7}x - \frac{40}{7}$ $\Rightarrow$ **$y = \frac{4}{7}x + \frac{648}{7}$**

These median lines meet at the centre of gravity. Solving our simultaneous equation (by hand or on calculator) we get that it is at **(62, 128)**

Excellence *requires showing higher level thinking. In this case using the nature of the parallelogram, and so* ***not*** *recalculating all the values by hand, but using that the median from B is the same line as the median from D, finding D by inspection rather than calculation, recognising that the medians for opposing corners will be parallel etc.*