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## Quadrilateral Properties – Co-ordinate Geometry Trial #3

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### Introduction

This assessment requires you to apply co-ordinate geometry methods using a specific shape on a co-ordinate plane.

The quality of your discussion and reasoning will determine your overall grade.

- Show your calculations.
- Use appropriate mathematical statements.
- Clearly communicate your strategy and method at each stage of your solution.

### Triangle Task

The quadrilateral **UVWX** has vertices **U** (-3, -2), **V** (-1, 4), **W** (14, 7) and **X** (12, 1).

Point **A** is the midpoint of line segment **UX**. Point **B** is the midpoint of line segment **VW**.

- Use co-ordinate geometry methods to investigate the properties of quadrilateral **UVWX**. Explain your reasoning.
- Show that line segment **AB** bisects line segment **UW**.
- Find a point **Y** on the line segment **VW** and a point **Z** on the line segment **UX** so that the quadrilateral **UVYZ** is a rhomboid.

Formula:  $y - y_1 = m(x - x_1)$

### Solutions:

Achieved – at least three different correct calculations (e.g. three out of length, midpoint, showing gradients are parallel, and equation of a line).

$$|UV| = \sqrt{(-1 - -3)^2 + (4 - -2)^2} = \sqrt{40} = 6.3245$$

$$|VW| = \sqrt{(14 - -1)^2 + (7 - 4)^2} = \sqrt{234} = 15.297$$

$$|WX| = \sqrt{(12 - 14)^2 + (1 - 7)^2} = \sqrt{40} = 6.3245$$

$$|UX| = \sqrt{(12 - -3)^2 + (1 - -2)^2} = \sqrt{234} = 15.297$$

$$\text{Point A} = \left(\frac{-3+12}{2}, \frac{-2+1}{2}\right) = (4.5, -0.5)$$

$$\text{Point B} = \left(\frac{-1+14}{2}, \frac{4+7}{2}\right) = (6.5, 5.5)$$

$$m_{UV} = \frac{4 - -2}{-1 - -3} = \frac{6}{2} = 3$$

$$m_{WX} = \frac{1 - 7}{12 - 14} = \frac{-6}{-2} = 3$$

$$m_{VW} = \frac{7 - 4}{14 - -1} = \frac{3}{15} = \frac{1}{5} = 0.2$$

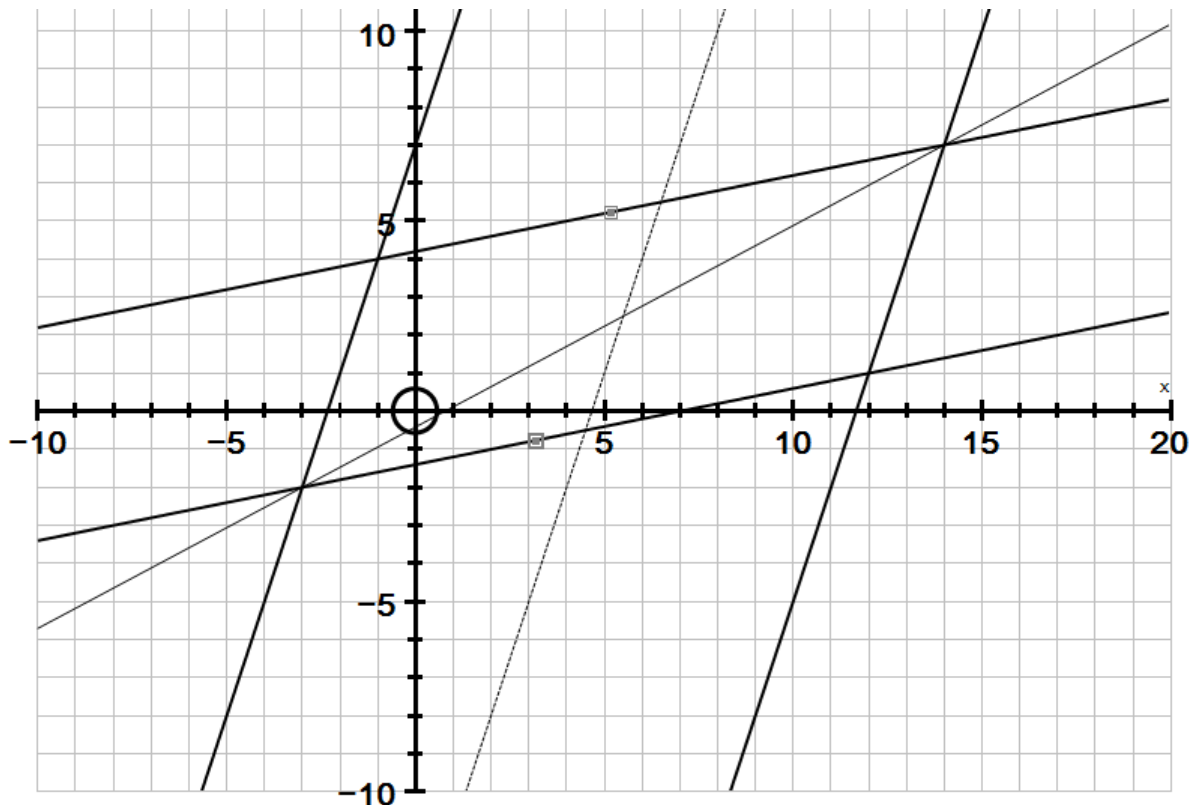
$$m_{UX} = \frac{1 - -2}{12 - -3} = \frac{3}{15} = \frac{1}{5} = 0.2$$

$$\text{Equation UV : } y - 4 = 3(x - -1) \Rightarrow y - 4 = 3x - 3 \quad \Rightarrow y = 3x + 7$$

$$\text{Equation VW : } y - 7 = 0.2(x - 14) \Rightarrow y - 7 = 0.2x - 2.8 \quad \Rightarrow y = 0.2x + 4.2$$

$$\text{Equation WX : } y - 1 = 3(x - 12) \Rightarrow y - 1 = 3x - 36 \quad \Rightarrow y = 3x - 35$$

$$\text{Equation UX : } y - 1 = 0.2(x - 12) \Rightarrow y - 1 = 0.2x - 2.4 \quad \Rightarrow y = 0.2x - 1.4$$



Merit

one of:

- The quadrilateral is a parallelogram because opposite sides have the same lengths.
- The quadrilateral is a parallelogram because opposite sides are parallel, which we see because they have the same slopes.
- The quadrilateral is not a rectangle, because the lines meeting are not perpendicular (e.g.  $m_{UV} \times m_{VW} \neq -1$ )

plus one of:

- The line AB bisects the line UW, which we see because their point of intersection is also the midpoint of UW.
- The line AB bisects the line UW, which we see because the distance from the point of intersection to U and also to W is the same.
- The line AB bisects the line UW, which we see because the distance from the point of intersection to U is half the length of UW.

$$m_{AB} = \frac{5.5 - -0.5}{6.5 - 4.5} = \frac{6}{2} = 3$$

$$m_{UW} = \frac{7 - -2}{14 - -3} = \frac{9}{17}$$

$$\text{Equation AB : } y - 5.5 = 3(x - 6.5) \Rightarrow y - 5.5 = 3x - 19.5 \quad \Rightarrow y = 3x - 14$$

$$\text{Equation UW : } y - 7 = \frac{9}{17}(x - 14) \Rightarrow y - 7 = \frac{9}{17}x - \frac{126}{17} \quad \Rightarrow y = \frac{9}{17}x + \frac{-7}{17}$$

Equation UW intersects Equation AB at I = (5.5, 2.5)

$$\text{Midpoint UW} = \left( \frac{-3+14}{2}, \frac{-2+7}{2} \right) = (5.5, 2.5)$$

$$|UI| = \sqrt{(-3 - 5.5)^2 + (-2 - 2.5)^2} = \sqrt{92.5} = 9.618$$

$$|WI| = \sqrt{(14 - 5.5)^2 + (7 - 2.5)^2} = \sqrt{92.5} = 9.618$$

$$|UW| = \sqrt{(-3 - 14)^2 + (-2 - 7)^2} = \sqrt{370} = 19.235$$

Excellence - Find either Y or Z correctly

$$|UV| = \sqrt{(-1 - -3)^2 + (4 - -2)^2} = \sqrt{40} = 6.3245$$

$$\text{so we need } |VW| = \sqrt{40} = \sqrt{(x - -1)^2 + (y - 4)^2}$$

$$\text{Squaring both sides, and } 40 = (x + 1)^2 + (y - 4)^2$$

$$\text{But we know Y lines on the line } y = 0.2x + 4.2 \text{ so we can say } 40 = (x + 1)^2 + (0.2x + 0.2)^2.$$

$$\text{Expanding } 40 = x^2 + 2x + 1 + 0.04x^2 + 0.08x + 0.04$$

$$\text{That rearranges to } 1.04x^2 + 2.08x - 38.96 = 0$$

The calculator gives the solution as 5.2017 (we ignore -7.2017, which is going the wrong way)

$$\text{Put into } y = 0.2x + 4.2 \text{ we get } y = 5.24034$$

By a similar calculation, or inspection that it shifted by the same as U→V (-2, -6)

$$Y = (5.20, 5.24) \text{ and } Z = (3.20, -0.76)$$