Basic for Algebraic Fractions

❶ Any number is converted to a fraction by putting it over 1.

$$5x = \frac{5x}{1}$$

❷ Multiplication is done by multiplying both denominators and numerators..

$$\frac{a}{x} \times \frac{b}{k} = \frac{a \times b}{x \times k} = \frac{ab}{xk}$$

The reverse is true, so you can separate out multiplication factors from the **top** line.

$$\frac{3x}{4} = \frac{3 \times x}{4 \times 1} = \frac{3}{4} \times \frac{x}{1} = \frac{3}{4}x$$

❸ Addition is done by adding the numerator but leaving the denominator the same.

$$\frac{y}{2x} + \frac{4}{2x} = \frac{y+4}{2x}$$

The reverse is true – a fraction with addition or subtraction on the **top** line can be split into two fractions, but this is **not** true for + or – in the denominator.

$$\frac{x+4}{5} = \frac{x}{5} + \frac{4}{5}$$

❹ The bottom line of a fraction can be rewritten in a new form by multiplying top and bottom by the same number. That number can also be an unknown.

$$\frac{x}{y} = \frac{3 \times x}{3 \times y} = \frac{3x}{3y} \quad \text{and} \quad \frac{x}{y} = \frac{x \times x}{x \times y} = \frac{x^2}{xy}$$

This technique allows us to get the same denominators, so they can be added.

$$\frac{2}{x} + \frac{3}{y} = \frac{2y}{xy} + \frac{3x}{xy} = \frac{2y+3x}{xy}$$

The fractions are not being changed, they are just rewritten in a new form.

❺ Any common **multiplication** factor can be cancelled out top and bottom.

$$\frac{5x}{25} = \frac{5 \times x}{5 \times 5} = \frac{x}{5} \quad \text{and} \quad \frac{k^2+3k+2}{4k+4} = \frac{(k+2)\cancel{(k+1)}}{4\cancel{(k+1)}} = \frac{k+2}{4}$$

*But you can only cancel out common factors, **you cannot cancel out additions**.*

$\frac{x+2}{x+3}$ *and* $\frac{x^2+2}{2x^2}$ *cannot be simplified, as they have no common multiplication factor.*

❻ The denominator of a fraction can be removed **in an equation** by multiplying it across.

$$\frac{x}{x+2} = \frac{3}{x+2} \quad \Rightarrow \quad x(x+2) = 3(x+3)$$

This is the only time denominators can be removed, except by cancelling top and bottom.
It is vital to recognise when you have an equation – and can do this – and when you can't.

❼ All negatives and subtractions should always be moved to be the numerator, to reduce errors – and particularly errors from double negatives.

$$\frac{5}{x^3} - \frac{3}{kx} = \frac{5}{x^3} + \frac{-3}{kx} \quad \text{and} \quad \frac{a}{-b} = \frac{-a}{b}$$