

## Networks #1 Cycling holiday

Susan lives in Town A in the centre of the local district. She is going to ride all the cycle tracks around for her holiday. The distances for each track (in kilometres) is given in the table:

	A						
B	12	B					
C	10	5	C				
D	12	–	10	D			
E	15	7	–	–	E		
F	9	15	–	–	13	F	
G	9	–	–	3	–	–	G
H	–	–	8	9	–	–	–

- (a) What is the quickest route between Town F and Town H?

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- (b) The local council has decided to upgrade the tracks. They can't do them all at first because of cost, but want to make sure each town has at least one upgraded path going to it. What is the least amount of kilometres they need to upgrade to do this?

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- (c) Discuss the possible routes so that Susan can cycle every track once, and only once. How does living in Town A affect this?

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- (d) Discuss how adding another track from Town A will affect Susan's ability to ride every track once and only once.

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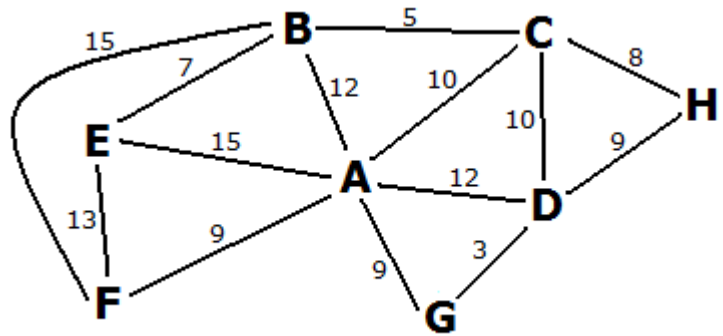


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## Answers: Networks #1 Cycling Holiday

The connected table looks like

(the network must connect as shown but can be drawn with the points and edges in any position)



- (a) What is the quickest route between Town F and Town H?

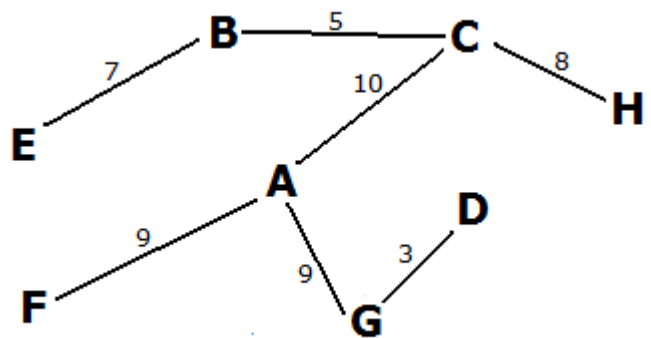
**F–A–C–H**

- (b) The local council has decided to upgrade the tracks. They can't do them all at first because of cost, but want to make sure each town has at least one upgraded path going to it. What is the least amount of kilometres they need to upgrade to do this?

**51 kilometres of track.**

**There are two ways of doing this with a choice of A–C or D–C making no difference.**

The minimum spanning tree involved is shown to the right (and must be explained, visually or by listing the edges in your answer).



- (c) Discuss the possible routes so that Susan can cycle every track once, and only once. How does living in Town A affect this?

**The original network has odd vertices only at E and F. So a route that goes over each track once must start at either E and end at F or vice versa.**

**Susan cannot start from Town A and ride such a route.**

- (d) Discuss how adding another track from Town A will affect Susan's ability to ride every track once and only once.

**If a new track from A goes to either E or F, then Susan will be able to ride every track once and only once starting from Town A, but cannot do so such that she returns to town A. (That would need yet another track from A to the other of E or F, or for the new track to link E and F instead.)**

**If the new track from A goes to anywhere other than E or F, then it will no longer be possible to ride a route over each track exactly once from any starting point because there will be more than two odd vertices.**