Sequences and Series Practice #1

$t_n = a + (n - 1)d$ $\qquad$ $t_n = a\,r^{\,n-1}$

$S_n = \frac{n}{2}[2a + (n-1)d]$ $\qquad$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $\qquad$ $S_\infty = \frac{a}{1 - r}$

1. Billy is stacking tins in a triangle for a supermarket display.

   Each row is one more than the one above.

   a) How many tins will he need to lay on the base to make the stack 12 rows high?

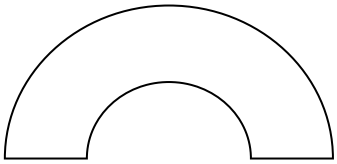
   b) How many rows high can he stack if he starts with 1000 tins?

2. For the pattern $t_n = 5n - 2$, what is the value of the first 200 terms added up?

3. Romans built their theatres in the form of semicircles.

   A archaeologist excavating one found it had 20 rows. The first row sat 80 people. Most of the other rows were too damaged to assess, but the 14th row sat 132 people.

   Estimate the total seating capacity of the theatre.



4. An internet start up company plans to sell 2,400 copies of its programs in the first year, and increase sales by 25% each year after that.

   a) How many copies is it projecting to sell in its 10th year?

   b) How many copies of the programs will it sell over the first twelve years?

5. Calculate the value of the series $1 + \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \ldots$

6. The population of a small Pacific island was 3,406 in 2010. A population loss of 10% per year can be assumed based on recent years. Estimate the population in 2003.

Answers: Sequences and Series Practice #1

1.a) How many tins will he need to lay on the base to make the stack 12 rows high?

$a = 1$, $d = +1$, want t_{12} $t_n = a + (n - 1)d = 1 + (12 - 1) \times 1$

He needs twelve tins along the bottom row

b) How many rows high can he stack if he starts with 1000 tins?

$a = 1$, $d = +1$, want $S_n < 1000$

$S_n = \frac{n}{2}[\ 2a + (n - 1)d]$ so $\frac{n}{2}[\ 2 \times 1 + (n - 1) \times 1] < 1000$

Solving gives 44.22. **He can stack up to 44 rows high** (must round down)

2. Calculate the value of: $\sum_{n=1}^{200} (5n - 2)$

$t_1 = 3$, $t_2 = 8$, $t_3 = 13$, so $a = 3$, $d = 5$, need S_{200}

$S_{200} = \frac{n}{2}[\ 2a + (n - 1)d] = \frac{200}{2}[\ 2 \times 3 + (200 - 1) \times 5] =$ **100,100**

3. Estimate the total seating capacity of the theatre.

The increase will be arithmetic (as circumference is proportional to diameter).

$a = 40$, $d = (132 - 80) \div (14 - 1) = 4$, need S_{20}

$S_{20} = \frac{n}{2}[\ 2a + (n - 1)d] = \frac{20}{2}[\ 2 \times 80 + (20 - 1) \times 4] =$ **2,360 people**

4. An internet start up company plans to sell 2,400 copies of its programs in the first year, and increase sales by 25% each year after that.

a) How many copies is it projecting to sell in its 10th year?

$a = 2400$, $r = 1.25$, $n = 10$ $t_{10} = a\, r^{\,n-1} = 2400 \times 1.25^{\,10-1}$ = **17,881 copies**

b) How many copies of the programs will it sell over the first twelve years?

$a = 2400$, $r = 1.25$, $n = 12$ $S_{12} = \frac{a(r^n - 1)}{r - 1} = \frac{2400 \times (1.25^{12} - 1)}{1.25 - 1}$ = **130,098 copies**

5. Calculate the value of the series $1 + \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \ldots$

$a = 1$, $r = 0.2$ or $\frac{1}{5}$, sum is infinite $S_\infty = \frac{a}{1 - r} = \frac{1}{1 - 0.2} =$ **1.25**

6. The population of a small Pacific island was 3,406 in 2010. A population loss of 10% per year can be assumed based on recent years. Estimate the population in 2003.

$r = 0.90$ (100% – 10%), 2003 = 1, so $n = 8$ (2010 – 2003 + 1), $t_8 = 3406$

$t_n = a\, r^{\,n-1}$ so $3406 = x \times 0.90^{8-1}$ **Population was approx 7121**

Achieved = Q1 a), Q2, Q4 a) and b). Merit = Q1 b) and Q5. Excellence = Q3 and Q6.