## Level 2 Trigonometry Harder #1

1. Calculate *a*, which is clearly **not** 36.3°



3. Calculate C



5. What is the value of *e*?



Two 4 m wires are attached to a pole.
They make an angle of 50° to the ground.

At ground level they are 120° apart.

What angle do they make with each other on the pole,  $\theta$ ?



2. The area is 245. Calculate *b*.



4. What is the area of this triangle?



6. Calculate *F*°



 Find two lengths that meet inside the circle so that the area of the triangle produced is 10 cm<sup>2</sup>



## Answers: Level 2 Trigonometry Harder #1

Most of the problems can be approached in more than one way, but the methods given here are usually the shortest.

Rounding errors will occur unless you carry all the decimal places.

1. 
$$\sin a = \frac{\sin 19^{\circ}}{22} \times 40 = 0.5919$$
  $a = \sin^{-1}(0.5919) = 36.29^{\circ}$   
However for any *x*, sin *x* = sin (180° - *x*) so sin 36.29° = sin 143.71° = 0.5919

So our angle here is 143.71°

. . . . .

3.

2. Calculate third side, x, from Area:  $245 = \frac{1}{2} \times x \times 22 \times \sin 130^\circ x = 29.075$ 

 $b^2 = 29.075^2 + 22^2 - 2 \times 29.075 \times 22 \times \cos 130^\circ = 2151.67$   $b = \sqrt{2151.67} = 46.39$ 



Using geometry the angles in the triangle to the right are 150° (on a line) and 10° (180° in triangle)

Using the Sine Rule on the right triangle:  $L = \frac{33}{\sin 10^{\circ}} \times \sin 20^{\circ} = 64.997$ Using simple trig on the left triangle:  $C = \sin 30^{\circ} \times 64.997 = 32.5$ 

4. Need an angle between sides. Call angle opposite 29 side  $y^{\circ}$ :

 $\sin y^{\circ} = \frac{\sin 109^{\circ}}{40} \times 29 = 0.6855 \qquad y^{\circ} = \sin^{-1}(0.6855) = 43.27^{\circ}$ Remaining angle is then  $x = 180^{\circ} - 43.27^{\circ} - 109^{\circ} = 27.72^{\circ}$ Now have angle between sides. Area =  $\frac{1}{2} \times 29 \times 40 \times \sin 27.72^{\circ} = 269.79$ 

5. Find the diagonal across -x

$$x^{2} = 20^{2} + 49^{2} - 2 \times 20 \times 49 \times \cos 85^{\circ} = 2630.17 \qquad \qquad x = \sqrt{2630.17} = 51.29$$

Now use that value to find angle between that side x and side e

$$\sin y^{\circ} = \frac{\sin 65^{\circ}}{51.29} \times 35 = 0.61852$$
  $A = \sin^{-1}(0.61852) = 38.21^{\circ}$ 

That leaves the other angle in the bottom triangle being  $180^{\circ} - 38.21^{\circ} - 65^{\circ} = 76.79^{\circ}$  $e^{2} = 51.29^{2} + 35^{2} - 2 \times 51.29 \times 35 \times \cos 76.79^{\circ} = 3035.2$   $e = \sqrt{3035.2} = 55.09$  6. Find the diagonal across -k

$$k^2 = 20^2 + 29^2 - 2 \times 20 \times 29 \times \cos 118 = 1785.59$$
  $k = \sqrt{1785.59} = 42.25$ 

Now we have two triangles, for which we know all the sides. We can do  $x^{\circ}$  in two parts

top 
$$\cos a^{\circ} = \frac{20^{2} + 42,25^{2} - 29^{2}}{2 \times 20 \times 42.25} = \frac{1344.06}{1690}$$
  $a^{\circ} = \cos^{-1}(\frac{1344.06}{1690}) = 37.3^{\circ}$   
bottom  $\cos b^{\circ} = \frac{40^{2} + 42,25^{2} - 28^{2}}{2 \times 40 \times 42.25} = \frac{2601.06}{3380}$   $b^{\circ} = \cos^{-1}(\frac{2601.06}{3380}) = 39.7^{\circ}$   
 $F^{\circ} = 37.3^{\circ} + 39.7^{\circ} = 77.0^{\circ}$ 

7. The wires are  $\cos 50^{\circ} \times 4 = 2.571$  metre out from the base of the pole The distance from one end to the other is 4.4533 m

( as 
$$d^2 = 2.571^2 + 2.571^2 - 2 \times 2.571 \times 2.571 \times \cos 120^\circ = 19.83$$
 )

Then we can find the angle  $\boldsymbol{\theta}$  as we now have all three sides of the triangle

$$\cos \theta = \frac{4^2 + 4^2 - 3.63 \ 4.453^2}{2 \times 4 \times 4} \text{ so } \theta = 67.65^{\circ}$$

8. There are various strategies that work.

example 1: assume one of the new sides is a length – let's say it is also 5 cm.

Then from the area we can calculate the angle between that side and the first side.

 $10 = \frac{1}{2} \times 5 \times 5 \times \sin A$ , so the angle is 53.13°.

The third side is then  $x^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 53.13$ , so is 4.472cm

example 2: assume one of the new sides at a given angle – let's say it is 50° – from the first line.

Then from the area we can calculate the side length of that side.

 $10 = \frac{1}{2} \times 5 \times x \times \sin 50$  so the side is 5.22 cm.

The third side is  $x^2 = 5^2 + 5.22^2 - 2 \times 5 \times 5.22 \times \cos 50$ , so is 4.324 cm

The similarity of the two calculations give confidence they are right, but they can also be checked by finding the other angles and seeing that they give the right area.

