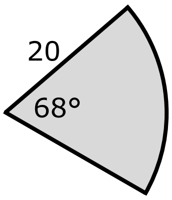
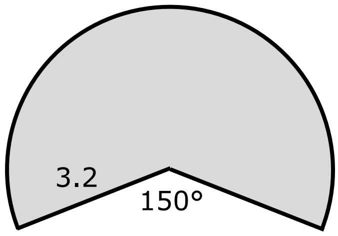
Level 2 Trigonometry Sectors and Segments #3

All curves shown are all parts of circles.

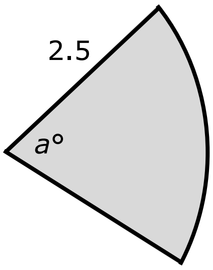
1. Calculate the shaded area shown:



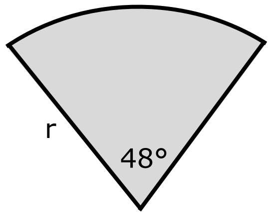
2. Calculate the perimeter of this shape



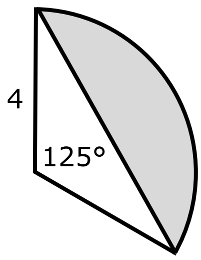
3. The shaded area is 5 m^2. What is a°?



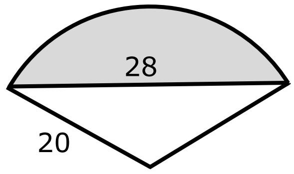
4. The perimeter is 25 m. What is r?



5. Calculate the shaded area

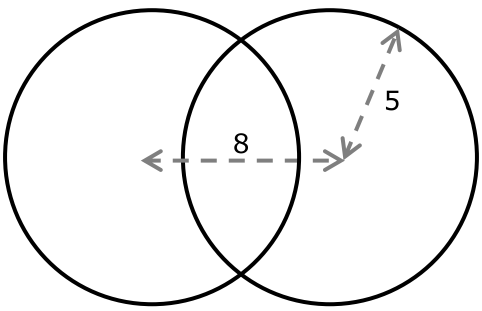


6. Calculate the area of the segment.



7. Two circles of radius 5 m are placed so that their centres are 8 m apart.

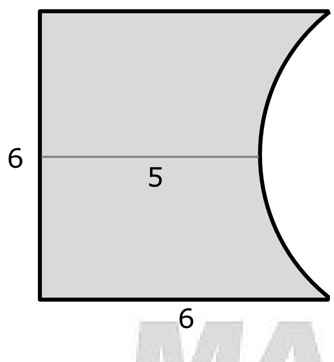
   What is the area of the overlap?



8. There is a square of side edges 6 cm.

   An arc is taken out of an entire side, so that the nearest that arc reaches to the other side is 5 cm.

   Find the shaded area remaining.



Answers: Level 2 Trigonometry Sectors and Segments #3

Rounding errors will occur unless you carry all the decimal places.

1. $A = \frac{68}{360} \times \pi \times 20^2 =$ **237.36**

   or

   $68° = 68 \times \frac{2\pi}{360} = 1.1868$ radians $\quad A = ½\, \theta\, r^2 = 0.5 \times 1.1868 \times 20^2 = 237.36$

2. The arc's angle is $360 - 150 = 210°$ so the arc length, $a = \frac{210}{360} \times \pi \times 2 \times 3.2 = 11.73$

   Add in the two radiuses, perimeter = **18.13**

   or

   $210° = 210 \times \frac{2\pi}{360} = 3.665$ rad $\quad p = r\theta + r + r = 3.665 \times 3.2 + 3.2 + 3.2 = 18.13$

3. $5 = f \times \pi \times 2.5^2$, where f is the fraction of the circle. $f = \frac{5}{\pi \times 2.5^2} = 0.2546$

   $0.2546 \times 360 =$ **91.67°**

   or

   $5 = ½\, \theta \times 2.5^2 \quad \Rightarrow \quad \theta =$ **1.6 radians**

4. $25 = [\, \frac{48}{360} \times \pi \times 2 \times r\,] + r + r = 25 \quad \Rightarrow \quad 25 = 2.84r \quad$ radius = **8.81**

   or

   $48° = 48 \times \frac{2\pi}{360} = 0.8378$ rad $\quad 25 = r\theta + r + r \Rightarrow 25 \times 2.8378 \quad r = 8.81$

5. Area sector $= \frac{125}{360} \times \pi \times 4^2 = 17.453$

   Area triangle $= ½ \times 4 \times 4 \times \sin(125) = 6.553$

   Shaded area = sector − triangle = $17.453 - 6.553 =$ **10.9**

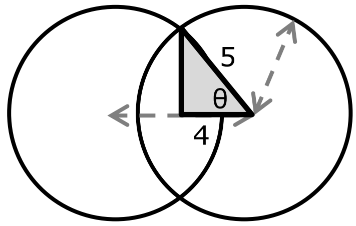
6. To find the angle: $\cos a° = \frac{20^2 + 20^2 - 28^2}{2 \times 20 \times 20} = \frac{16}{400} \quad a° = \cos^{-1}(\frac{16}{400}) = 87.71°$

   Area sector $= \frac{87.71}{360} \times \pi \times 20^2 = 306.16$

   Area triangle $= ½ \times 20 \times 20 \times \sin(87.71) = 199.84$

   segment = sector − triangle = $306.16 - 199.84 =$ **106.3**

7. A right angle triangle 4 wide and 5 hypotenuse can be drawn:



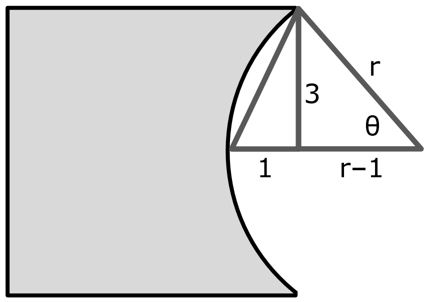
$\theta = \cos^{-1}(\frac{4}{5}) = 36.87°$ and we need $2\theta = 73.74°$

The overlap is two segments, one from each centre..

(doing them like Q6) Area sector $= \frac{73.74}{360} \times \pi \times 5^2 = 16.09$

Area triangle = $\frac{1}{2} \times 5 \times 5 \times \sin(73.74) = 12$ Overlap $= 2 \times (16.09 - 12) =$ **4.18 cm²**

8. A right angle triangle can be drawn



The height is half the square height = 3

The width is one less than the radius as the circle pokes 1 cm into the square.

Using Pythagoras $r^2 = 3^2 + (r - 1)^2$ giving r = 5

$\theta = \sin^{-1}(\frac{3}{5}) = 36.87°$ and we need $2\theta = 73.74°$

Area sector $= \frac{73.74}{360} \times \pi \times 5^2 = 16.09$

Area triangle = $\frac{1}{2} \times 5 \times 5 \times \sin(73.74) = 12$

Area of segment = 16.09 – 12 = 4..09

Square's area = 36, less the segment taken out = 36 – 4.09 = **31.9 cm²**

You can also calculate the angle at the centre of the circle using

angle on 1 by 3 triangle $= \tan^{-1}(\frac{3}{1}) = 71.565°$, doubled is 143.13°

angle at centre is double angle at edge, so angle at centre = 286.26

But the angle we want is on the other side, so angle we need = 360 – 286.26 = 73.74°

That gives θ = 36.87°, and using the triangle above r = 3 ÷ sin(36.87) = 5

[or you can use $6^2 = r^2 + r^2 - 2 \times r \times r \times \cos(73.74)$ so r = 5]

Then as above.