Calculus Fractions Practice #1

Solve:

1. $\dfrac{3}{x+2} = 4$

2. $\dfrac{2}{x} + \dfrac{3}{x^2} = 7$

3. $\dfrac{2}{x+1} + \dfrac{1}{x} = 8$

4. $\dfrac{x}{x-8} - \dfrac{24}{x+2} = 3$

Write for y in terms of x, as a simplified single fraction:

5. $\dfrac{3}{x+1} + \dfrac{1}{y} = 1$

6. $\dfrac{x}{3+\frac{2}{y}} = 5$

7. $\dfrac{2}{x+y} + \dfrac{1}{x} = 8$

8. $\dfrac{1}{xy} + \dfrac{1}{x} = 4$

Rearrange and simplify to give these to their simplest possible two line fractions (note, they become very simple fractions in most cases):

9. $\dfrac{\frac{a}{b^3}}{\frac{a^3}{b}}$

10. $\dfrac{15+8x+x^2}{35+2x-x^2}$

11. $\dfrac{1+\frac{2}{x}}{2+\frac{5}{x}}$

12. $\dfrac{1}{2+\frac{5}{2+\frac{1}{x}}}$

Answers: Calculus Fractions Practice #1

In general to solve you need one fraction on both sides, so in questions 3 and 4 the fraction terms are combined by adding/subtracting. But question 2 clearly has the form of a quadratic, so the denominators are cancelled as one much easier step (you could combine terms, but it is much slower).

1. $\frac{3}{x+2} = 4$ $\quad 3 = 4(x + 2)$ $\quad 3 = 4x + 8$ $\quad \mathbf{x = {}^{-}1.25}$

2. $\frac{2}{x} + \frac{3}{x^2} = 7$ $\quad 2x + 3 = 7x^2$ $\quad 0 = 7x^2 - 2x - 3$ $\quad \mathbf{x = 0.183 \text{ or } {}^{-}0.527}$

3. $\frac{2}{x+1} + \frac{1}{x} = 8$ $\quad \frac{2x+x+1}{x(x+1)} = 8$ $\quad 3x + 1 = 8x^2 + 8x$ $\quad \mathbf{x = 0.159 \text{ or } {}^{-}0.784}$

4. $\frac{x}{x-8} - \frac{24}{x+2} = 3$ $\quad \frac{x(x+2) - 24(x-8)}{(x-8)(x+2)} = 3$ $\quad x^2 + 2x - 24x + 192 = 3x^2 - 18x - 48$

$0 = 2x^2 + 4x - 240$ $\quad \mathbf{x = 10 \text{ or } {}^{-}12}$

To write y in terms of x you need to get the y as the only variable on one side of the equation as a first step.

5. $\frac{3}{x+1} + \frac{1}{y} = 1$ $\quad \frac{1}{y} = 1 - \frac{3}{x+1}$ $\quad \frac{1}{y} = \frac{x+1-3}{x+1}$ $\quad \mathbf{y = \frac{x+1}{x-2}}$

6. $\frac{x}{3+\frac{2}{y}} = 5$ $\quad x = 15 + \frac{10}{y}$ $\quad \frac{1}{y} = \frac{x-15}{10}$ $\quad \mathbf{y = \frac{10}{x-15}}$

7. $\frac{2}{x+y} + \frac{1}{x} = 8$ $\quad \frac{2}{x+y} = \frac{8x}{x} - \frac{1}{x}$ $\quad x + y = \frac{2x}{8x-1}$ $\quad \mathbf{y = \frac{3x - 8x^2}{8x-1}}$

8. $\frac{1}{xy} + \frac{1}{x} = 4$ $\quad \frac{1}{xy} = \frac{4x}{x} - \frac{1}{x}$ $\quad xy = \frac{x}{4x-1}$ $\quad \mathbf{y = \frac{1}{4x-1}}$

Anything on the bottom line is a division, so turn a fractional denominator to a multiplication by its inverse. If the bottom line is a partial fraction, then combine terms first to make it a single fraction. Cancel common factors, by factorising quadratics if required.

9. $\frac{\frac{a}{b^3}}{\frac{a^3}{b}}$ $\quad = \frac{a}{b^3} \times \frac{b}{a^3}$ $\quad = \frac{ab}{a^3b^3}$ $\quad = \mathbf{\frac{1}{a^2b^2}}$

10. $\frac{15 + 8x + x^2}{35 + 2x - x^2}$ $\quad = \frac{x^2 + 8x + 15}{-(x^2 - 2x - 35)}$ $\quad = \frac{(x+3)(x+5)}{-(x-7)(x+5)}$ $\quad = \mathbf{\frac{x+3}{7-x}}$

11. $\frac{1+\frac{2}{x}}{2+\frac{5}{x}}$ $\quad$ (top and bottom $\times\, x$ cancels fractions) $\quad = \mathbf{\frac{x+2}{2x+5}}$

12. $\frac{1}{2+\frac{5}{2+\frac{1}{x}}}$ $\quad = \frac{1}{\frac{4x+2}{2x+1} + \frac{5x}{2x+1}}$ $\quad = \frac{1}{\frac{9x+2}{2x+1}}$ $\quad = \mathbf{\frac{2x+1}{9x+2}}$