Calculus Fractions Practice #3

Solve:

1. $\frac{5}{x^2} + x = 8$

2. $\frac{\frac{x}{8}}{\frac{4}{x}} = 2$

3. $\frac{3}{x-2} - 7 = \frac{5}{x-1}$

4. $\frac{\frac{5}{x-1}}{\frac{14}{x+1}} = 2$

Write for y in terms of x, as a simplified single fraction:

5. $\frac{4}{x/y} = 1$

6. $\frac{1}{xy} + \frac{1}{x^2y} = 5$

7. $\frac{\frac{1}{y}}{x - \frac{1}{y}} + \frac{3}{x} = 2$

8. $\frac{xy}{x+y} = 4$

Rearrange and simplify to give these to their simplest possible two line fractions (note, they become very simple fractions in most cases):

9. $\frac{\frac{1}{y} + \frac{2}{x}}{\frac{1}{x} + \frac{2}{y}}$

10. $\frac{5}{2 + \frac{x}{2+x}}$

11. $\frac{1}{x^3 - x} + \frac{x}{1 - x^2}$

12. $\frac{1 - \frac{x^2y^2}{z^2}}{1 + \frac{xy}{z}}$

Answers: Calculus Fractions Practice #3

There are multiple ways of answering these questions, but usually one is much easier than the others.

1. $\frac{4}{x^2} + x = 3$ $\quad 4 + x^3 = 3x^2$ $\quad 0 = x^3 - 3x^2 + 4$ $\quad \boldsymbol{x = 2 \text{ or } -1}$

2. $\frac{\frac{x}{8}}{\frac{4}{x}} = 2$ $\quad \frac{x}{8} = 2 \times \frac{4}{x}$ $\quad x^2 = 64$ $\quad \boldsymbol{x = \pm 8}$

3. $\frac{3}{x-2} - 7 = \frac{5}{x-1}$ $\quad \frac{3-7(x-2)}{x-2} = \frac{5}{x-1}$ $\quad \frac{-7x+17}{x-2} = \frac{5}{x-1}$

   $(-7x + 17)(x - 1) = 5(x - 2)$ $\quad 0 = 7x^2 - 19x + 7$ $\quad \boldsymbol{x = 2.275 \text{ or } 0.4396}$

4. $\frac{\frac{10}{x-1}}{\frac{7}{x+1}} = 2$ $\quad \frac{10}{x-1} = \frac{14}{x+1}$ $\quad 10(x + 1) = 14(x - 1)$ $\quad \boldsymbol{x = 6}$

Flipping both sides of a fraction is often a quick way to simplify the process, but this only applies if you have an equation – do not apply it to an inequation (>, < etc).

5. $\frac{4}{x/y} = 1$ $\quad 4 = \frac{x}{y}$ $\quad 4y = x$ $\quad \boldsymbol{y = \frac{x}{4}}$

6. $\frac{1}{xy} - \frac{1}{x^2y} = 5$ $\quad \frac{x-1}{x^2y} = 5$ $\quad \frac{x-1}{x^2} = 5y$ $\quad \boldsymbol{y = \frac{x-1}{5x^2}}$

7. $\frac{\frac{1}{y}}{x - \frac{1}{y}} + \frac{3}{x} = 2$ $\quad \frac{1}{xy-1} = \frac{2x-3}{x}$ $\quad xy - 1 = \frac{x}{2x-3}$ $\quad \boldsymbol{y = \frac{3x-3}{x(2x-3)}}$

8. $\frac{xy}{x+y} = 4$ $\quad xy = 4x + 4y$ $\quad y(x - 4) = 4x$ $\quad \boldsymbol{y = \frac{4x}{x-4}}$

9. $\frac{\frac{1}{y} + \frac{2}{x}}{\frac{1}{x} + \frac{2}{y}}$ $\quad$ (multiply by $\frac{xy}{xy}$) $\quad = \boldsymbol{\frac{x+2y}{y+2x}}$

10. $\frac{5}{2 + \frac{x}{2+x}}$ $\quad = \frac{5(2+x)}{2(2+x) + \frac{x(2+x)}{2+x}}$ $\quad = \frac{10+x}{4+2x+x}$ $\quad = \boldsymbol{\frac{x+10}{3x+4}}$

It is very important to remember the relationship $a - b = -(b - a)$:

11. $\frac{1}{x^3 - x} + \frac{x}{1-x^2}$ $\quad = \frac{1}{x(x^2-1)} + \frac{-x^2}{x(x^2-1)}$ $\quad = \frac{1-x^2}{x(x^2-1)}$ $\quad = \boldsymbol{\frac{-1}{x}}$

12. $\frac{1 - \frac{x^2y^2}{z^2}}{\frac{1}{z} + \frac{xy}{z^2}}$ $\quad = \frac{z^2 - x^2y^2}{z + xy}$ $\quad = \frac{(z+xy)(z-xy)}{z+xy}$ $\quad = \boldsymbol{z - xy}$