Calculus Quadratics Practice #3

Solve, by completing the square:

1. $6x^2 - 13x = {}^{-}8$

2. $2x^2 + x - 28 = 0$

3. $28x^2 + 3x - 40 = 0$

Solve, by completing the square, to give solutions in exact form (a + √b for surds):

4. $x^2 - 4x - 11 = 0$

5. $2x^2 - 10x + 1 = 0$

6. $x^2 + 13x + 4 = 0$

Solve, using the quadratic formula, to give solutions in exact form (a + √b for surds):

7. $x^2 + 11x + 5 = 0$

8. $2x^2 - 4x - 7 = 0$

9. $5x^2 - 9x + 2 = 0$

For what values of k do the following equations have real solutions?

10. $kx^2 - 5x + 3 = 0$

11. $x^2 + 4kx - 3k = 0$

Show that for every (real) value of k, that there are real solutions:

12. $x^2 + 4kx + k^2 = 0$

Answers: Quadratics Practice #3

Solve, by completing the square:

1.	$6x^2 - 13x = {}^{-}8$	$(x - \frac{13}{12})^2 - (\frac{13}{12})^2 = \frac{-8}{6}$	$(x - \frac{13}{12}) = \pm\sqrt{\frac{361}{144}}$	$x = \frac{1}{2}$ or $\frac{8}{3}$
2.	$2x^2 + x - 28 = 0$	$(x - 0.25)^2 = 0.25^2 + 14$	$(x - 0.25) = \sqrt{14.0625}$	$x = 3.5$ or ${}^{-}4$
3.	$28x^2 + 3x - 40 = 0$	$(x - \frac{3}{56})^2 - (\frac{3}{56})^2 - \frac{40}{28} = 0$	$(x - \frac{3}{56}) = \pm\sqrt{\frac{4489}{3136}}$	$x = \frac{8}{7}$ or $\frac{5}{4}$

Solve, by completing the square, to give solutions in exact form (a + √b for surds):

4.	$x^2 - 4x - 11 = 0$	$(x - 2)^2 - 2^2 - 11 = 0$	$(x - 2) = \sqrt{15}$	$x = 2 \pm \sqrt{15}$
5.	$2x^2 - 10x + 1 = 0$	$(x - 2.5)^2 - 2.5^2 + 0.5 = 0$	$(x - 2.5) = \pm\sqrt{5.75}$	$x = 2.5 \pm \sqrt{5.75}$
6.	$x^2 + 13x + 4 = 0$	$(x + 6.5)^2 - 6.5^2 + 4 = 0$	$(x + 6.5)^2 = \sqrt{38.25}$	$x = {}^{-}6.5 \pm \sqrt{38.25}$

Solve, using the quadratic formula, to give solutions in exact form (a + √b for surds):

7.	$x^2 + 11x + 5 = 0$	$\frac{-11 \pm \sqrt{11^2 - 4 \times 1 \times 5}}{2 \times 1}$	${}^{-}5.5 \pm \frac{\sqrt{101}}{2}$	$x = {}^{-}5.5 \pm \sqrt{25.25}$
8.	$2x^2 - 4x - 7 = 0$	$\frac{--4 \pm \sqrt{4^2 - 4 \times 2 \times -7}}{2 \times 2}$	$\frac{4}{4} \pm \frac{\sqrt{72}}{4}$	$x = 1 \pm \sqrt{4.5}$
9.	$5x^2 - 9x + 2 = 0$	$\frac{--9 \pm \sqrt{9^2 - 4 \times 5 \times 2}}{2 \times 5}$	$0.9 \pm \frac{\sqrt{41}}{10}$	$x = 0.9 \pm \sqrt{0.41}$

For what values of k do the following equations have real solutions?

10.	$kx^2 - 5x + 3 = 0$	$(x - \frac{5}{2k})^2 - (\frac{5}{2k})^2 + \frac{3}{k} = 0$	$(x - \frac{5}{2k}) = \pm\sqrt{(\frac{25}{4k^2} - \frac{3}{k})}$	$k \leq \frac{25}{12}$
	or $b^2 - 4ac \geq 0$	$({}^{-}5)^2 - 4 \times k \times 3 \geq 0$	$25 - 12k \geq 0$	$k \leq \frac{25}{12}$
11.	$x^2 + 4kx - 3k = 0$	$(x + 2k)^2 - (2k)^2 - 3k = 0$	$(x + 2k) = \pm\sqrt{(4k^2 + 3k)}$	$k \geq 0$ or $k \leq \frac{-3}{4}$
	or $b^2 - 4ac \geq 0$	$(4k)^2 - 4 \times 1 \times 3k \geq 0$	$4k(4k + 3) \geq 0$	$k \geq 0$ or $k \leq \frac{-3}{4}$

Show that for every (real) value of k, that there are real solutions:

12. $x^2 + 4kx + k^2 = 0$ $\quad (x + 2k)^2 - (2k)^2 + k^2 = 0$ $\quad (x + 2k) = \pm\sqrt{(3k^2)}$ $\quad x = {}^{-}2k + \pm\sqrt{(3k^2)}$

$(3k^2)$ is always positive ⇒ $\sqrt{(3k^2)}$ is always a real number ⇒ ${}^{-}2k + \pm\sqrt{(3k^2)}$ is always real

or $\Delta = b^2 - 4ac = (4k)^2 - 4 \times 1 \times k^2 = 12k^2$. k^2 is >0 for every value of k, so $12k^2$ is always positive. Since the discriminant, Δ, is always positive, there are always real solutions.