Y13 Extension – 3 × 3 Equations

1. If I have the equation $4x + 3y + z = 12$. What does that look like as a physical representation in 3 dimensions?

2. Show the nature of the system of equations:

   $4x + 3y - z = 11$ $\quad$ $3x - y + 2z = 9$ $\quad$ $7x - 11y + 12z = 21$

3. Find k so that the system of equations below is consistent:

   $9x + y + 3z = -3$ $\quad$ $x + 6y + 2z = 6$ $\quad$ $15x + 24 = ky$

4. Write the solutions for a and b in the following system of equations in terms of c and find a solution.

   $2a + \frac{b}{2} + \frac{c}{12} = 2$ $\quad$ $a - 3b + \frac{c}{3} = 5$ $\quad$ $-6a - 60b + 5c = 66$

5. Find the equation of the parabola that passes through (2, 3.23), (3, 5.33) and (5, 8.33).

6. Find the integer solutions to the system of equations:

   $7x + 2z = 20 + y$ $\quad$ $x + y + z = 10$ $\quad$ $7y + 4z = 40 + x$

Answers: Y13 Extension – 3 × 3 Equations

1.

A plane, passing through (0, 0, 12), (0, 4, 0) and (3, 0 , 0),

A plane, perpendicular to the vector $\begin{pmatrix}4\\3\\1\end{pmatrix}$ passing through (1, 1, 5)

Or similar explanations.

2.

Solving $4a + 3b = 7$ and $3a + -1b = -11$ gives us $a = -2$ and $b = 5$

This gives us our multiples, so that eqn ③ = −2 × eqn ① + 5 × eqn ②

$$\begin{array}{llr} & -2 \times ① = & -8x + -6y + 2z = -22 \\ + & 5 \times ② & 15x - 5y + 10z = 45 \\ + & -1 \times ③ & \underline{-7x + 11y - 12z = -21} \\ & & 0 + 0 + 0 = 2 \end{array}$$

Since $0 \neq 2$, **our system must be inconsistent**.

3.

Rewrite as: ① $9x + y + 3z = -3$ ② $x + 6y + 2z = 6$ ③ $15x - ky = -24$

Solving $9a + 1b = 15$ and $3a + -2b = 0$ gives us $a = 2$ and $b = -3$

This gives us our multiples, so that eqn ③ = 2 × eqn ① + −3 × eqn ②

Using our multiples, $-k = 2 \times 1 + -3 \times 6 = -16$, so $\boldsymbol{k = 16}$

4.

Multiply the equations out to get rid of fractions, giving us:

① $24a + 6b + c = 24$ ② $3a - 9b + c = 15$ ③ $-6a - 60b + 5c = 66$

to get rid of b 3×① + 2×② ⇒ $78a + 5c = 102$ ⇒ ④ $c = 20.4 - 15.6a$

to get rid of a −1×① + 8×② ⇒ $-78b + 7c = 96$ ⇒ ⑤ $c = \frac{96}{7} + \frac{78}{7}b$

If we set $a = 0$, then $c = 20.4$ (from ④) so then $b = 0.6$ (from ⑤)

A solution is ($a = 0$, $b = 0.6$, $c = 20.4$)

5.

A parabola is $a\,x^2 + b\,x + c = d$ which we can use to write equations

$4a + 2b + c = 3.23$, $9a + 3b + c = 5.33$ and $25a + 5b + c = 8.33$

Which solves on the calculator to give: $\boldsymbol{y = 0.2x^2 + 3.1x + 2.17}$

6.

① $7x - y + 2z = 20$ ② $x + y + z = 10$ ③ $-x + 7y + 4z = 40$ ⑦⑧⑨

to get rid of x ① − 7×② ⇒ $-8y - 5z = -50$ ⇒ ④ $8y + 5z = 50$

to get rid of y ① + ② ⇒ ⑤ $8x + 3z = 30$

to get rid of z ① − 2×② ⇒ $5x - 3y = 0$ ⇒ ⑥ $5x = 3y$

Eqn ⑥ is the useful one, as it has the least terms. It limits us (for integer solutions) to situations where we are dealing with a multiple of 15 because otherwise $5x \neq 3y$.

We can generalise that to the situation where $x = 3n$ and $y = 5n$ for n any integer. We can then substitute this into eqn ② so that $z = 10 - 3n - 5n$.

The solutions follow the scheme: $\boldsymbol{x = 3n, y = 5n, z = 10 - 8n \ \forall n \in \mathbb{Z}}$