Calculus Trigonometry Practice #1

1. A voltmeter is placed across an electric circuit which is fluctuating with time.

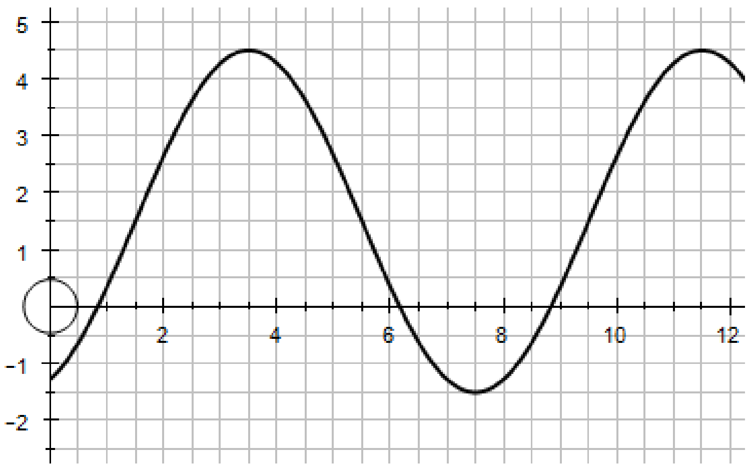
   The maximum reading on the voltmeter is 12 volts, and the lowest reading is 11.4 volts.

   The current fluctuates 50 times per second.

   Construct a model for the voltage, V, relative the time in milliseconds, t.

   Use that model to show how long in each cycle the voltage is above 11.8 volts.

2. Write four different equations for the graph shown to the right (i.e. not just shifted one period each time).



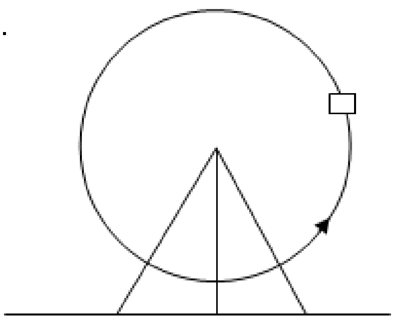
3. A Ferris wheel spins so that one revolution takes 50 minutes.

   The passengers' lowest point is 1.8 metres off the ground.

   The highest point is 22.4 metres high.

   A passenger gets in at the lowest point at the wheel's start time of 9:00 a.m.

   Construct a general solution to the times the passenger is above 18 metres.



Answers: Calculus Trigonometry Practice #1

Solutions may be done with different trig equations from those shown.

1. Amplitude = (12 – 11.4) ÷ 2 = 0.3   Midpoint = (12 + 11.4) ÷ 2 = 11.7

   Period = 20 milliseconds.   The start point is not given so set first peak at t = 0.

   V = 0.3 cos $\left(\frac{2\pi}{20}t\right)$ + 11.7   (NB: sine graph will be exactly the same.)

   Solving for V = 11.8 gives solutions of 3.918 and 16.082.

   But these solutions span the trough, not the crest, so moving the first one forward 20, gives 23.918 and 16.082 ms (which is symmetrical across the crest at 20 ms).

   Each cycle is above 11.8 V for 23.918 – 16.082 = **7.84 milliseconds**.

2. Amplitude = (4.5 – $^{-}$1.5) ÷ 2 = 3   Midpoint = (4.5 + $^{-}$1.5) ÷ 2 = 1.5

   Period = 11.5 – 3.5 = 8 (from peak to peak).   Shift = 3.5 to peak for cos graph

   ***y* = 3 cos $\left(\frac{2\pi}{8}(t-3.5)\right)$ + 1.5**   ***y* = 3 sin $\left(\frac{2\pi}{8}(t-1.5)\right)$ + 1.5**

   ***y* = 1.5 – 3 cos $\left(\frac{2\pi}{8}(t-7.5)\right)$**   ***y* = 1.5 – 3 sin $\left(\frac{2\pi}{8}(t-5.5)\right)$**

3. Amplitude = (22.4 – 1.8) ÷ 2 = 10.3   Midpoint = (22.4 + 1.8) ÷ 2 = 12.1

   Period= 50 minutes.   Set t = 0 at 00:00, so trough at t = 540 and peaks at 540 + 25

   ***H* = 10.3 cos $\left(\frac{2\pi}{50}(t-565)\right)$ + 12.1**   or *H* = $^{-}$10.3 cos $\left(\frac{2\pi}{50}(t-540)\right)$ + 12.1

   Solving for H = 18 gives $t = \cos^{-1}\left(\frac{5.9}{10.3}\right) \times \frac{50}{2\pi} + 565$ = **572.6** (9:33 a.m.), on down slope.

   Up-slope is the other side of the peak, *t* = 565 – (565 – 572.6) = **557.4** (9:17 a.m.)

   The solutions are 7.6 minutes either side of the peaks, and the peak are repeated every fifty minutes, starting with the first at 540 + 25, so the general solution is:
   507.4 + 50*n* < t < 522.6 + 50*n*   where $n \in \mathbb{N}$ (i.e. *n* is an integer starting at one) representing the number of revolutions from 9:00, and *t* is minutes after midnight.