Practice for L3 Distributions #1 – Fishy Stuff

Question One

A sample of 580 Brazilian toadfish (*Porichthys porosissimus*) was weighed, accurate to the nearest 100 g.

The weights showed a normal distribution, with mean of 5.1 kg and a standard deviation of 0.55 kg.

a) What is the estimated probability an adult toadfish from the sample would be measured at 6 kg or over?

b) A scientist wants to study the heaviest fish only in a follow-up study. What weight should he use to select only the top 20% of all toadfish?

c) What is the probability that a catch of **20** toadfish will exceed 100 kg?

Question Two

Coliform bacteria are randomly distributed in a river at an average concentration of 1.2 per 20cc of water.

a) If 20cc of water is taken at random, what is the chance that the sample contains no coliform bacteria?

b) If 10cc of water is taken at random, what is the chance that the sample contains at least 2 coliform bacteria?

c) What size sample would give a probability of zero coliforms of 0.5? *Show your equations.*

Question Three

Sue runs a business which takes people out to swim with dolphins in summer. She makes a trip every day of the week. She knows that she finds suitable pods 85% of the time.

a) Using a binomial distribution, calculate the probability that in a week (i.e. seven trips) that she find dolphins six or seven times.

b) Using a binomial distribution, calculate the probability that she finds dolphins at least four times in every week in February? (Assume February has four weeks.)

c) What assumptions have to be made to use a binomial distribution in parts a) and b) ? Comment on how likely these assumptions are to be valid.

Answers: Practice for L3 Distributions #1

Q1 a) Normal distribution with a lower bound of 6.0 gives **P(weight ≥ 6 kg) = 0.0509**. **= A**

However should apply a continuity correction, as anything 5.95 kg up counts as 6.0.

Use x > 5.95 kg, with μ = 5.1 and σ = 0.55

P(weight ≥ 6 kg) = 0.0611 **= M**

b) Inverse normal, with area 0.2 to right, or 0.8 to left. Calculator gives = 5.56. **= A**

However we need to round to the nearest 100 g, and rounding down gives > 20%

Top 20% will exceed 5.6 kg **= M**

c) E(X + Y) = E(X) + E(Y), so in this case we add twenty means = 20 × 5.1

Mean weight of 20 fish = 102 kg

Var(X + Y) = Var(X) + Var(Y), so in this case we add 20 variances = 20 × 0.55^2

Standard deviation of 20 fish = √6.05 = 2.46.

If we assume weighing to nearest 100g we need 99.95 kg as our lower bound.

P(20 fish ≥ 100 kg) = 0.7979 or 0.7919 without correction **= E**

Q2 a) Poisson distribution with λ = 1.2. Need value for x = 0.

P(zero coliforms) = 0.3012 **= A**

b) Poisson distribution with λ = 1.2 ÷ 2 = 0.6 for 10 cc selections.

Cumulative for x = 2+, which is the same as for 1 – P(0 or 1) = 1 – 0.8781.

P(2+ coliforms) = 0.1219 **= M**

c) P(X = x) = $\frac{e^{-\lambda}\lambda^x}{x!}$ In this case our x = 0, our probability is = 0.5 and we need λ

0.5 = $\frac{e^{-\lambda}\lambda^0}{0!}$ = $\frac{e^{-\lambda}1}{1}$ = $e^{-\lambda}$. Log_e both sides: ln(0.5) = ln($e^{-\lambda}$) = – λ ln(e) = – λ = M

λ = 0.693. To get a sample size with that lambda we solve $\frac{0.693}{1.2} = \frac{x}{20}$

Our sample should be 11.55 cc **= E**

Q3 a) p = 0.85, n = 7, x = 6 or 7. P(6) = 0.3960 + P(7) = 0.3206 = **P(6+) = 0. 7166 = A**

b) p = 0.85, n = 7, x = 4 – 7. Find this from = 1 – P(0 – 3) P(4+) = 0. 9879. = A

Four consecutive = 0.9879^4 so **P(four weeks at 4+) = 95.25%** **= M**

c) Have a only two possible results (dolphins or not) and a fixed number of trials.

Need to assume that the probability is constant for each trial, but some times of year may be more likely than other. It may be that it is relatively constant through summer though.

Need to assume that the probability on consecutive days is independent, which is unlikely, since if dolphins are there one day they are more likely to be the next. **= E**