Practice for Merit L3 Probability #1

Question One

A boys hockey team of 11 players is randomly chosen from *x* number of boys.

Two of the boys are twins.

Calculate the probability that neither of the twins are in the team chosen for the day.

Question Two

A random variable X takes even number values from 2 to 2*n* inclusive. These values occur with equal probability.

Find the mean and variance of this distribution.

Hint $\sum_{k=1}^{n} 2k = n(n+1)$ *and* $\sum_{k=1}^{n} (2k)^2 = \frac{2n(n+1)(2n+1)}{3}$

Question Three

Harry is organising a team to represent his school in the local Maths competition.

A team consists of three students. They are to be randomly chosen from five girls and four boys.

Find the probability that the team selected has a majority of girls.

Question Four

Peter has three brightly coloured bags.

The blue bag has *x* black and *y* white marbles.

The red bag has *z* black and *w* white marbles.

One marble from each of the other two bags is randomly selected and placed into the empty green bag.

Find the probability that the green bag will have two marbles that are the same colour.

Explain your reasoning carefully.

Question Five

Andrew, Billy and Cameron roll a normal six-sided dice in turn, starting with Andrew.

The first to roll a six wins.

What is the probability that Andrew wins?

Question Six

A diner can choose three scoops of ice-cream from five flavours (raspberry, strawberry, lime, chocolate and vanilla). Repeat flavours are allowed, but there is no order of scoops.

How many different selections are there?

Answers: Practice for Merit L3 Probability #1

1. The number of ways the 11 members of the squad of x can be picked is : ${}^{x}C_{11}$

   using the formula sheet: $= \dfrac{x!}{11!(x-11)!}$

   If two members not in team then team chosen from (x – 2) instead of x.

   The number of ways of picking a team of 11 from (x – 2) is: ${}^{x-2}C_{11}$

   using the formula sheet: $= \dfrac{(x-2)!}{11!(x-13)!}$

   $$\text{P(twins not in team)} = \frac{\text{number of ways without them}}{\text{total possible ways}} = \frac{\frac{(x-2)!}{11!(x-13)!}}{\frac{x!}{11!(x-11)!}}$$

   $$\frac{(x-2)!}{11!(x-13)!}\,\frac{11!(x-11)!}{x!} = \frac{11!(x-11)!(x-2)!}{11!(x-13)!x!} = \frac{11!}{11!}\,\frac{(x-11)(x-12)(x-13)!}{(x-13)!}\,\frac{(x-2)!}{x(x-1)(x-2)!}$$

   $$= \frac{(x-11)(x-12)}{x\,(x-1)}$$

   Alternatively: chance first not in team is however many places are left over after the 11 are picked, out of x places $= \frac{x-11}{x}$. The chance the second is also not in the team is however many places are left after the 11 are picked, minus the one for his brother $= \frac{x-12}{x-1}$. Multiplying them gives $= \dfrac{(x-11)(x-12)}{x\,(x-1)}$

2.

x	2	4	6	...	2n
P(X = x)	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	...	$\frac{1}{n}$

$$E(X) = 2 \times \frac{1}{n} + 4 \times \frac{1}{n} + 6 \times \frac{1}{n} \ldots + n \times \frac{1}{n} = 2 \times (1 + 2 + 3 \ldots + n)\ \frac{1}{n}$$

$$= \frac{2n(n+1)}{2n} = n + 1$$

$$E(X^2) = 2^2 \times \frac{1}{n} + 4^2 \times \frac{1}{n} + 6^2 \times \frac{1}{n} \ldots + n^2 \times \frac{1}{n} = (2^2 + 4^2 + 6^2 \ldots + (2n)^2)\ \frac{1}{n} =$$

$$= \frac{2n(n+1)(2n+1)}{3n} = \frac{2(n+1)(2n+1)}{3}$$

$$Var(X) = E(X^2) - [E(X)]^2 = \frac{2(n+1)(2n+1)}{3} - (n+1)^2 = \frac{n^2 - 1}{3}$$

3. Majority girls means two or three.

P(3 girls) = $\frac{{}^5C_3}{{}^9C_3}$ = 0.1190

P(2 girls, 1 boy) = $\frac{{}^5C_2 \times {}^4C_1}{{}^9C_3}$ = 0.4762

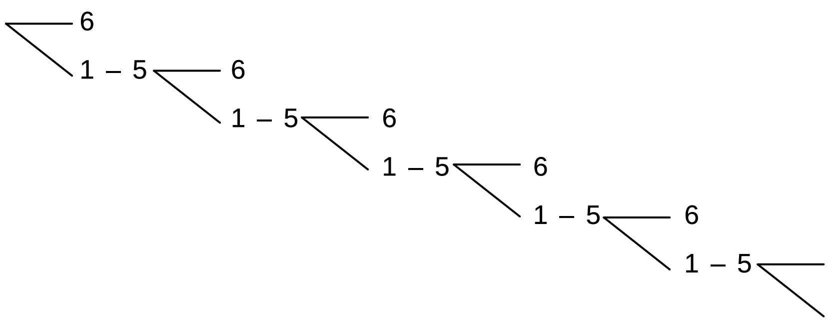
Added gives P(majority girls) = 0.5952 (= $\frac{25}{42}$)

4. P(2 black) = $\frac{x}{x+y} \times \frac{z}{z+w} = \frac{xz}{(x+y)(z+w)}$.

P(2 white) = $\frac{y}{x+y} \times \frac{w}{z+w} = \frac{yw}{(x+y)(z+w)}$.

P(2 same) = P(2 black) + P(2 white) = $\frac{xz+yw}{(x+y)(z+w)}$.

5. roll 1 roll 2 roll 3 roll 4 roll 5



A wins	1/6
B wins	5/6 × 1/6
C wins	$(5/6)^2$ × 1/6
A wins	$(5/6)^3$ × 1/6
B wins	$(5/6)^4$ × 1/6
etc	etc

P(A wins) = $1/6 + (5/6)^3 \times 1/6 + (5/6)^6 \times 1/6$ …

P(B wins) = $5/6 \times 1/6 + (5/6)^4 \times 1/6 + (5/6)^7 \times 1/6$ …

P(C wins) = $(5/6)^2 \times 1/6 + (5/6)^5 \times 1/6 + (5/6)^8 \times 1/6$ …

P(C) = P(B) × 5/6 = P(A) × $(5/6)^2$

So 1 = P(A) + 5/6 × P(A) + 25/36 × P(A)

P(A) = 36/91 = 0.3956 *alternatively* = $\frac{a}{1-r} = \frac{\frac{1}{6}}{1-(\frac{5}{6})^3} = \frac{36}{91}$

6. There are ${}^5C_3$ = 10 ways of getting three different flavours.

There are ${}^5C_2$ = 10 ways of getting two different scoops, but we need to × 2, because it matters which one is the double.

There are ${}^5C_1$ = 5 ways of getting only one flavour.

10 + 10 × 2 + 5 = 35

Alternatively, we could line the flavours up, and the diner makes a choice to skip to the next. There are 3 scoops and four points between the flavours. e.g. ⊗|||⊗⊗⊗.

That gives ${}^7C_3$ = 35.