Practice for Merit L3 Probability #3

Question One

P(E) = 0.7. P(D) = 0.2). E and D are independent:

a) what is $P(E \cup E') = ?$

b) what is $P(E \cap E') = ?$

c) what is $P(E \cup D) = ?$

d) what is $P(E' \cap D') = ?$

Question Two

Steven fancies Courtney. If they and five other friends go to the movies and sit in an order decided by a random selection of tickets, what is the chance Steven will be sitting by Courtney?

Question Three

James has a normal 6-sided dice (marked 1, 2, 3, 4, 5, 6) and a 6-sided dice marked 2, 2, 3, 3, 4, 6.

If he rolls both ten times and adds the twenty scores, what would he expect to get as a total?

Question Four

In each test five coins are tossed at a time, and the number of heads counted.

If $P_r = P(R = r \text{ heads})$, what is $\sum_{r=0}^{3} P_r$?

Question Five

How many different ways can ten players be split into two teams of five?

Question Six

One timetable option at a school allows students to take exactly **two** of Chemistry, Physics, Hort and ICT.

There are 39 taking Physics, of whom 12 also take ICT. There are 34 taking Chemistry, of whom 25 also take Physics. There are 36 taking ICT, of whom 8 also take Chemistry.

What is the probability that a randomly chosen student in that timetable option is taking Hort?

Answers: Practice for Merit L3 Probability #3

1. a) $P(E \cup E') = P(E)$ <u>or</u> its opposite $= 1$

   b) $P(E \cap E') = P(E)$ <u>and</u> its opposite $= 0$

   c) $P(E \cup D) = P(E) + P(D) - P(E \cap D) = 0.7 + 0.2 - 0.7 \times 0.2 = 0.76$

   d) $P(E' \cap D') = P(E') \times P(D') = 0.3 \times 0.8 = 0.24$

2. There are 7! ways seven people can sit where order matters.

   If Steven and Courtney are a pair, then there are 6! the group can be arranged, × 2 because it could go SC or CS.

   $$\frac{6! \times 2}{7!} = \frac{2}{7}$$

   alternatively, there is a 2/7 chance Courtney is sitting on either end, with 1/6 Steven is beside her + 5/7 she is in the middle with 2/6 Steven is beside her.

   $$\frac{2}{7} \times \frac{1}{6} + \frac{5}{7} \times \frac{2}{6} = \frac{2}{7}$$

3.

Score	1	2	3	4	5	6	Totals
$P(X = x)$	1/6	1/6	1/6	1/6	1/6	1/6	1
E(X) calc	1/6	2/6	3/6	4/6	5/6	6/6	3.5

Score	1	2	3	4	5	6	Totals
$P(X = x)$	0/6	2/6	2/6	1/6	0/6	1/6	1
E(X) calc	0/6	2/6	6/6	4/6	0/6	6/6	3

   Dice rolls are independent, so $E(10X + 10Y) = 10E(X) + 10E(Y)$

   $= 10 \times 3.5 + 10 \times 3 = 65$

4. Drawing a tree, or similar, gives $P_0 = \frac{1}{32}$, $P_1 = \frac{5}{32}$, $P_2 = \frac{10}{32}$, $P_3 = \frac{10}{32}$, $P_4 = \frac{5}{32}$, $P_5 = \frac{1}{32}$

   $$\sum_{r=0}^{3} P_r = P_0 + P_1 + P_2 + P_3 = \frac{26}{32} = 0.8125$$

5. ${}^{10}C_5 = 252$, but that gives a set of mirror images, so that is two times too much.

   $252 \div 2 = 126$ ways

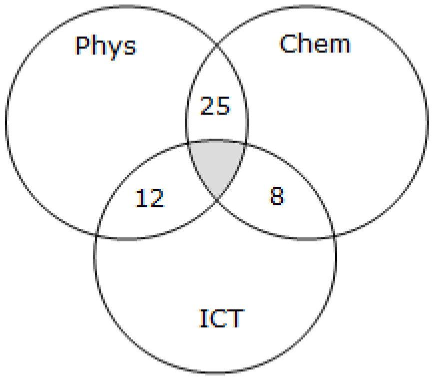
   alternatively, take one player and designate whichever team he is in the first team.

   That leaves ${}^{9}C_4 = 126$ ways.

6. There are a few ways of answering, one of the easiest involves using Venn diagrams.

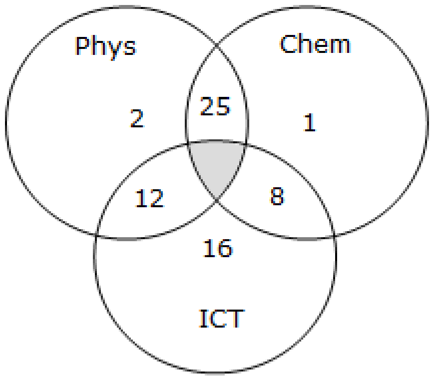
Put in the known ones doing only Chemistry, Physics and ICT. Start with the given overlaps among those three.

Note the inner area is grey as no-one can take three.



Now we can use the total that take Physics, Chemistry and ICT to fill in the outer circles.

e.g. 39 take Physics, of which 25 are crossed with Chem and 12 with 12. That leaves 2.



The outer areas represent the students doing Hort and Physics (2), Hort and Chemistry (1) and Hort and ICT (16) = 19 total taking Hort.

Adding all the numbers gives the total students in that option
= 2 + 25 + 1 + 12 + 8 + 16 = 64

probability a student in the line takes Hort =n19 out of 64 = 29.7%